EG2008 Revision Notes Week 9 Common Test

2008 S2

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Topics

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Integration by Parts

- There are 3 ways to apply integration by parts

1. General formula $\int u\,dv = uv - \int v\,du$

2. Tabular form (certain functions : x^n sin (ax+b), x^n cos (ax+b), x^n e^{ax+b} where n, a, and b are constants)

3. Special Integration (products of certain functions: e^{ax}, sin (ax+b), cos (ax+b) where a and b are constants)

Method 1: Integration by Parts using General Formula

- General formula

$$\int u\,dv = uv - \int v\,du$$

- Steps:
  1. Let $u = f(x)$, and $dv = g(x)dx$
  2. Differentiate u wrt x and find du
  3. Integrate $dv = g(x)\,dx$ to find v
  4. Substitute u, v, du back to the formula

Method 2: Integration by Parts using Tabular Form

- Apply to certain functions : $x^n \sin(ax+b)$, $x^n \cos(ax+b)$, $x^n e^{ax+b}$ where n, a, and b are constants

$$\int u\,dv = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots + (-1)^n \int u^{n-1} v_{n-1}\,dx$$

- Steps:
  1. Make a table with 2 columns; differentiate u and integrate *dv*
  2. Multiply *u* with v_1, *u*' with v_2, *u*'' with v_3 and so on; alternate the '+' or '-' sign
  3. Substitute back to the formula

Integration by Parts (Tabular Form)

Step 1:

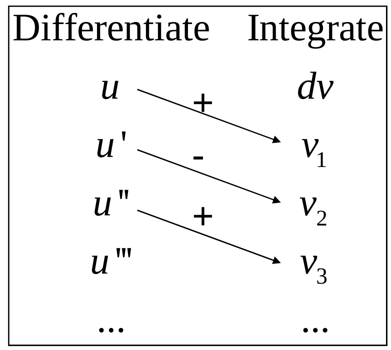
Make a table with 2 columns; differentiate *u* (until 0) and integrate *dv*

Differentiate	Integrate
u	dv
u'	v_1
u''	v_2
u'''	v_3
...	...

Integration by Parts (Tabular Form)

<u>Step 2:</u>

Multiply u with v_1, u' with v_2, u'' with v_3 and so on; alternate the '+' or '-' sign



Integration by Parts (Tabular Form)

<u>Step 3:</u>

Substitute back to the formula

$$\int u\,dv = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots + (-1)^n \int u^{n-1} v_{n-1}\,dx$$

Method 3: Integration by Parts using Special Integration

- Apply to product of certain functions : e^{ax}, sin (ax+b), cos (ax+b) where a and b are constants)

$$\int fg\,dx = \frac{fg' - f'g}{u - v} + C$$

where $f'' = vf$ and $g'' = ug$ and $u \neq v$

- <u>Steps</u>:
  1. Make a table with 2 columns; differentiate both f and g twice
  2. Identify the values of v and u by comparing $f'' = vf$ and $g'' = ug$
  3. If $u - v \neq 0$, then $\int fg\,dx = \dfrac{fg' - f'g}{u - v} + C$

Integration by Parts (Special Integration)

<u>Step 1:</u>

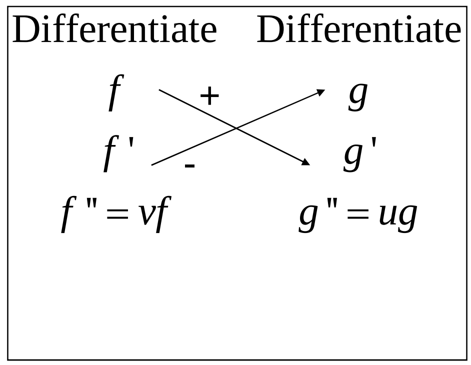
Make a table with 2 columns; differentiate both *f* and *g* twice

Differentiate		Differentiate
f	+	g
f'	-	g'
f''		g''

Integration by Parts (Special Integration)

Step 2:

Identify the values of v and u by comparing $f'' = vf$ and $g'' = ug$



Integration by Parts (Special Integration)

Step 3:

If $u - v \neq 0$, then substitute the values of fg', $f'g$, u and v into the formula

$$\int fg\,dx = \frac{fg' - f'g}{u - v} + C$$

The Summation Notation (∑)

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$$

$$\sum_{i=1}^{n} C = C + C + C + \ldots + C = nC$$

$$\sum_{i=1}^{n} (a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

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The Arithmetic Progressions (AP)

- *a* – first term; *n* – term number; *l* – last term; *d* – common difference ($d = T_2 - T_1$);
- General Term : $T_n = a + (n-1)d$
- Sum of first n terms (2 formulae):

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[a + l\right]$$

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The Geometric Progressions (GP)

- *a* – first term; *n* – term number;
  r – common ratio ($r = T_2/T_1$)
- General Term : $T_n = ar^{n-1}$
- Sum of first n terms :

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{if } r<1$$

$$S_n = \frac{a(r^n-1)}{r-1} \quad \text{if } r>1$$

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Binomial Expansion

- (Finite) If n is a positive integer, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots + x^n$$

- (Infinite) If n is any value, and $-1 < x < 1$, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

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Maclaurin Series

- If the derivate of the function exist at $x = 0$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^n(0)}{n!}x^n + \ldots$$

$$= f(0) + \sum_{n=1}^{\infty} \frac{f^n(0)}{n!}x^n$$

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Steps to find Fourier Series [$f(t)$ with $\omega = 2\pi/T$]

1. Find the period of $f(t)$, T
2. Choose a suitable d value ($d = -T/2$ or $d = 0$)
3. Find the Fourier coefficients

$$a_n = \frac{2}{T}\int_{d}^{d+T} f(t)\cos n\omega t\,dt \quad (n = 0, 1, 2, \ldots)$$

$$b_n = \frac{2}{T}\int_{d}^{d+T} f(t)\sin n\omega t\,dt \quad (n = 1, 2, \ldots)$$

4. Substitute the value of a_0, a_n, and b_n into

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

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Benefits of knowing whether *f*(*t*) is even function (pg 239)

- If *f*(*t*) is an **even** function, then the Fourier series consists of the constant term and the cosine terms only

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t \qquad b_n = 0$$

- where

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega t dt = \frac{4}{T}\int_{0}^{T/2} f(t)\cos n\omega t dt \quad \text{for} \quad n = 0,1,2,3\ldots$$

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Benefits of knowing whether *f*(*t*) is odd function

- If *f*(*t*) is an **odd** function, then the Fourier series consists of the sine terms only

$$f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t$$

where $a_0 = 0$ and $a_n = 0$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega t dt = \frac{4}{T}\int_{0}^{T/2} f(t)\sin n\omega t dt$$

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Fourier Series – Points to Note

- In general

1. $\cos(n\pi) = \cos(-n\pi) = (-1)^n \quad \text{for } n = 1, 2, 3, 4, \ldots$
2. $\sin(n\pi) = 0 \quad \text{for all } n$
3. $\cos(2n\pi) = 1 \quad \text{for all } n$
4. $\sin(2n\pi) = 0 \quad \text{for all } n$

Fourier Transform for Non-Periodic Waves

- A non-periodic waveform is a signal which varies with time.
- Use Fourier Transform to transform a time-varying signal to a frequency varying signal

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$