

EG2008 Revision Notes

Final Examination

1

Topics

- Integration by Parts (slide 4 -10)
- Summation Notation (slide 11)
- AP/GP (slide 12-13)
- Binomial Expansion (slide 14)
- Maclaurin Series (slide 15)
- Fourier Series (slide 16 – 19)
- Fourier Transform (slide 20)

2

Topics

□ Probability and Statistics

- Probability (slide 21-24)
- Discrete Distribution/Binomial/Poisson (slide 25 - 27)
- Normal Distribution (slide 28 - 29)
- Sampling Distribution (slide 30)
- Interval Estimation (slide 31)

3

Integration by Parts (Tabular Form)

Step 1:

Make a table with 2 columns; differentiate u (until 0) and integrate dv

Differentiate	Integrate
u	dv
u'	v_1
u''	v_2
u'''	v_3
...	...

4

Integration by Parts (Tabular Form)

Step 2:

Multiply u with v_1 , u' with v_2 , u'' with v_3 and so on; alternate the '+' or '-' sign

Differentiate		Integrate
u	$+$	dv
u'	$-$	v_1
u''	$+$	v_2
u'''	$-$	v_3
\dots		\dots

5

Integration by Parts (Tabular Form)

Step 3:

Substitute back to the formula

$$\int u dv = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \dots + (-1)^n \int u^{n-1} v_{n-1} dx$$

6

Method 3: Integration by Parts using Special Integration

- Apply to product of certain functions : e^{ax} , $\sin(ax+b)$, $\cos(ax+b)$ where a and b are constants)

$$\int fgdx = \frac{fg' - f'g}{u - v} + C$$

where $f'' = vf'$ and $g'' = ug'$ and $u \neq v$

- Steps:

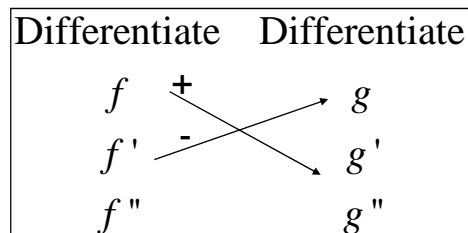
- Make a table with 2 columns; differentiate both f and g twice
- Identify the values of v and u by comparing $f'' = vf'$ and $g'' = ug'$
- If $u - v \neq 0$, then $\int fgdx = \frac{fg' - f'g}{u - v} + C$

7

Integration by Parts (Special Integration)

Step 1:

Make a table with 2 columns; differentiate both f and g twice

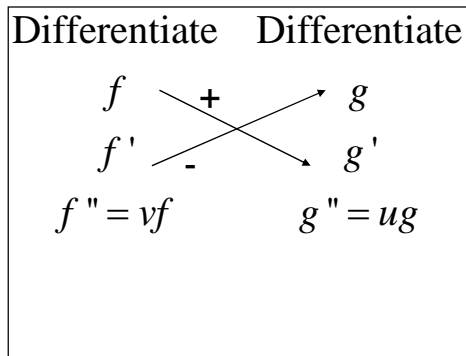


8

Integration by Parts (Special Integration)

Step 2:

Identify the values of v and u by comparing
 $f'' = vf$ and $g'' = ug$



9

Integration by Parts (Special Integration)

Step 3:

If $u - v \neq 0$, then substitute the values
of fg' , $f'g$, u and v into the formula

$$\int fgdx = \frac{fg' - f'g}{u - v} + C$$

10

The Summation Notation (Σ)

$$\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n$$

$$\sum_{i=1}^n C = C + C + C + \dots + C = nC$$

$$\sum_{i=1}^n (a_i \pm b_i) = \sum_{i=1}^n a_i \pm \sum_{i=1}^n b_i$$

$$\sum_{i=1}^n ca_i = c \sum_{i=1}^n a_i$$

11

The Arithmetic Progressions (AP)

- ▣ a – first term; n – term number; l – last term; d – common difference
($d = T_2 - T_1$);
- ▣ General Term : $T_n = a + (n-1)d$
- ▣ Sum of first n terms (2 formulae):

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[a + l]$$

12

The Geometric Progressions (GP)

- a – first term; n – term number;
 r – common ratio ($r = T_2/T_1$)
- General Term : $T_n = ar^{n-1}$
- Sum of first n terms :

$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{if } r < 1$$

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{if } r > 1$$

13

Binomial Expansion

- (Finite) If n is a positive integer, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots + x^n$$

- (Infinite) If n is any value, and $-1 < x < 1$, then

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

14

Maclaurin Series

- If the derivate of the function exist at $x = 0$

$$\begin{aligned} f(x) &= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^n(0)}{n!}x^n + \dots \\ &= f(0) + \sum_{n=1}^{\infty} \frac{f^n(0)}{n!}x^n \end{aligned}$$

15

Steps to find Fourier Series [$f(t)$ with $\omega = 2\pi/T$]

1. Find the period of $f(t)$, T
2. Choose a suitable d value ($d = -T/2$ or $d = 0$)
3. Find the Fourier coefficients

$$a_n = \frac{2}{T} \int_d^{d+T} f(t) \cos n\omega t dt \quad (n = 0, 1, 2, \dots)$$

$$b_n = \frac{2}{T} \int_d^{d+T} f(t) \sin n\omega t dt \quad (n = 1, 2, \dots)$$

4. Substitute the value of a_0 , a_n , and b_n into

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

16

Benefits of knowing whether $f(t)$ is even function (pg 239)

- If $f(t)$ is an **even** function, then the Fourier series consists of the constant term and the cosine terms only

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t \quad b_n = 0$$

- where

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos n\omega t dt = \frac{4}{T} \int_0^{T/2} f(t) \cos n\omega t dt \quad \text{for } n = 0, 1, 2, 3, \dots$$

17

Benefits of knowing whether $f(t)$ is odd function

- If $f(t)$ is an **odd** function, then the Fourier series consists of the sine terms only

$$f(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t$$

where $a_0 = 0$ and $a_n = 0$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin n\omega t dt = \frac{4}{T} \int_0^{T/2} f(t) \sin n\omega t dt$$

18

Fourier Series – Points to Note

□ In general

1. $\cos(n\pi) = \cos(-n\pi) = (-1)^n$ for $n = 1, 2, 3, 4, \dots$
2. $\sin(n\pi) = 0$ for all n

19

Fourier Transform for Non-Periodic Waves

- A non-periodic waveform is a signal which varies with time.
- Use Fourier Transform to transform a time-varying signal to a frequency varying signal

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

20

Definition of Probability

- m – number of outcomes from a Sample Sapce (experiment)
- n – number of outcomes from an event, E
- $P(E) = m/n$
 - $0 \leq P(E) \leq 1$
 - $P(E) + P(E') = 1$

21

Probability of Compound Event

- If 2 events A and B are **independent**, then
 - $P(A \cap B) = P(A) \times P(B)$
 - \cap (intersection) associates with the word 'and'
- If 2 events A and B are **mutually exclusive**, then
 - $P(A \cup B) = P(A) + P(B)$
 - \cup (union) associates with the word 'or'
- Additional Rule
 - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

22

Probability Tree

- Guidelines to draw probability tree:
 - Define stages involved in an experiment
 - For each branch, the probability should add up to 1
 - Multiply the probabilities along the branches to compute the final probability

23

Conditional Probability

- Given 2 events A and B, the conditional probability of A, assuming B has already occurred, is defined as
 - $P(A|B) = P(A \cap B) / P(B)$
 - $P(A \cap B) = P(A|B) \times P(B)$or
 - $P(B|A) = P(B \cap A) / P(A) = P(A \cap B) / P(A)$
 - $P(A \cap B) = P(B|A) \times P(A)$

24

Discrete Probability Distribution

- A discrete probability distribution can be expressed in terms of a table or a formula, in which $\sum_{\text{all } x} P(X = x) = 1$

- Expected value of X (average value)

$$\mu = E(X) = \sum_{\text{all } x} xP(X = x)$$

- Variance of X $\sigma^2 = \text{VAR}(X) = E(X - \mu)^2$
 $= E(X^2) - [E(X)]^2$

25

Special Discrete Distribution -- Binomial Distribution

- If the probability that an experiment results in a successful outcome is p and the probability that the outcome is a failure is q , where $q = 1 - p$, and if X is the random variable 'the number of successful outcomes in n independent trials', then the probability distribution of X is given by

$$X \sim B(n, p)$$

$$P(X = k) = {}^n C_k p^k q^{n-k} \quad k = 0, 1, 2, \dots, n$$

$$E(X) = np$$

$$\text{Var}(X) = npq$$

26

Special Discrete Distribution -- Poisson Distribution

- If X is the random variable represents 'the number of outcomes occurring in a given time interval or specified region', where μ is the **average number** of occurrences in a given time interval or specified region, then the probability distribution of X is given by $X \sim P_o(\lambda)$

$$P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad k = 0, 1, 2, \dots$$

$$E(X) = \text{Var}(X) = \lambda$$

27

Continuous Random Variable

- If X is a continuous random variable, then

$$\int_{-\infty}^{\infty} P(X) dx = 1$$

- i.e. the area below the curve $P(X)$ and above the x -axis represents the probability

$$P(a \leq X \leq b) = P(a < X < b)$$

$$\text{i.e. } P(X = a) = P(X = b) = 0$$

28

Special Continuous Distribution -- Normal Distribution

- For a normal r.v. X

$$X \sim N(\mu, \sigma^2)$$

$$P(a \leq X \leq b) = \int_a^b f(x) dx$$

$$E(X) = \mu$$

$$\text{VAR}(X) = \sigma^2$$

- Standardization of normal variable

Transform $X \sim N(\mu, \sigma^2)$
to $Z \sim N(0,1)$ (Standard Normal)

$$\text{through } Z = \frac{X - \mu}{\sigma}$$

29

The Central Limit Theorem

- If we draw random samples of size n from an infinite or a large population ($n \geq 25$) with mean μ and variance σ^2 , then the sampling distribution of the sample mean \bar{X} is approximated to a normal distribution with mean $\mu_{\bar{X}} = E(\bar{X}) = \mu$

and standard deviation $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$. Hence \bar{X}
can be standardized to

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

30

Interval Estimation of μ

□ 2 types

- σ^2 known, $(1 - \alpha)100\%$ confidence interval for

$$\mu = \bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

- σ^2 unknown, small sample ($n < 25$), $(1 - \alpha)100\%$ confidence interval for

$$\mu = \bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$$